

Pre-Calculus

12

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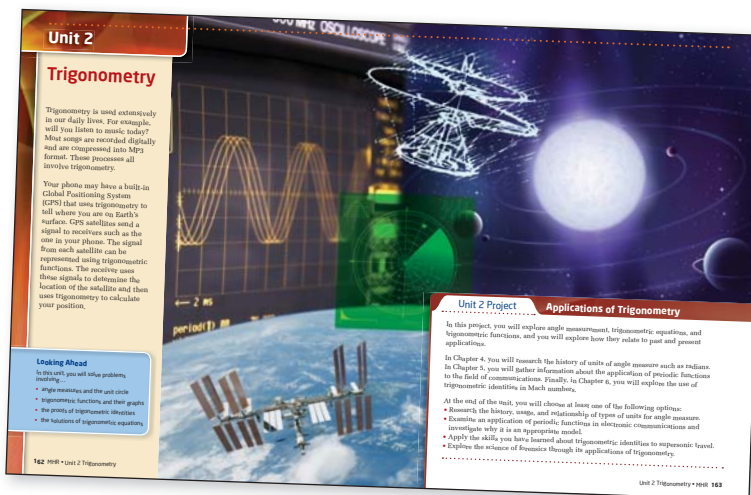
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A Tour of Your Textbook

Unit Opener

Each unit begins with a two-page spread. The first page of the **Unit Opener** introduces what you will learn in the unit. The **Unit Project** is introduced on the second page. Each **Unit Project** helps you connect the math in the unit to real life using experiences that may interest you.



Project Corner boxes throughout the chapters help you gather information for your project. Some **Project Corner** boxes include questions to help you to begin thinking about and discussing your project.

The **Unit Projects** in Units 1, 3, and 4 provide an opportunity for you to choose a single **Project Wrap-Up** at the end of the unit.

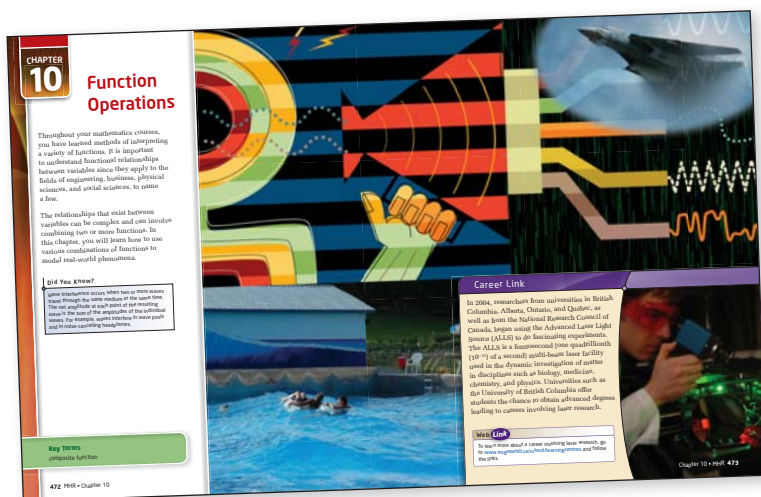
The **Unit Project** in Unit 2 is designed for you to complete in pieces, chapter by chapter, throughout the unit. At the end of the unit, a **Project Wrap-Up** allows you to consolidate your work in a meaningful presentation.

Chapter Opener

Each chapter begins with a two-page spread that introduces you to what you will learn in the chapter.

The opener includes information about a career that uses the skills covered in the chapter. A **Web Link** allows you to learn more about this career and how it involves the mathematics you are learning.

Visuals on the chapter opener spread show other ways the skills and concepts from the chapter are used in daily life.



Three-Part Lesson

Each numbered section is organized in a three-part lesson: Investigate, Link the Ideas, and Check Your Understanding.

Investigate

- The **Investigate** consists of short steps often accompanied by illustrations. It is designed to help you build your own understanding of the new concept.

- The **Reflect and Respond** questions help you to analyse and communicate what you are learning and draw conclusions.

Link the Ideas

- The explanations in this section help you connect the concepts explored in the **Investigate** to the **Examples**.

- The **Examples** and worked **Solutions** show how to use the concepts. The Examples include several tools to help you understand the work.
 - Words in green font help you think through the steps.
 - Different methods of solving the same problem are sometimes shown. One method may make more sense to you than the others. Or, you may develop another method that means more to you.

- Each Example is followed by a **Your Turn**. The Your Turn allows you to explore your understanding of the skills covered in the Example.

- After all the Examples are presented, the **Key Ideas** summarize the main new concepts.


3.1
Characteristics of Polynomial Functions

Focus on:

- analyzing polynomial functions
- analyzing polynomial functions

A cross-section of a honeycomb has a pattern with one hexagon surrounded by six more hexagons. Surrounding these is a third ring of 12 hexagons, and so on. The number of hexagons in a honeycomb when the graph of the function, $f(x)$, models the total x is the number of steps. Thus, you can use the graph of the function to solve questions about the honeycomb pattern.

A quadratic function that models this pattern will be discussed later in this section.



Did You Know?

Edwin Hubble is known as the "Father of Cosmology." He discovered that galaxies are moving away from Earth and that galaxies are moving away from each other in the universe.

Investigate Graphs of Polynomial Functions

Method 1: Graph each set of functions on a different set of coordinate axes using graphing technology. Sketch the results.

Type of Function	Set A	Set B	Set C	Set D
Linear	$f(x) = 2x - 3$	$f(x) = 3x - 1$	$f(x) = x + 1$	$f(x) = -x + 2$
Quadratic	$f(x) = x^2 - 2x + 3$	$f(x) = x^2 - 4x + 2$	$f(x) = x^2 + 2x + 1$	$f(x) = x^2 - 1$
Cubic	$f(x) = x^3 - 2x^2 + 3x - 1$	$f(x) = x^3 - 4x^2 + 2x + 1$	$f(x) = x^3 + 2x^2 + 1$	$f(x) = x^3 - 1$

Method 2: Compare the graphs and write each set of functions in terms of x and y .

and behaviour:

- degree of the function in one variable, x
- number of terms
- leading coefficient
- constant of integration

Check: Use the degree of the function to check the degree of the graph.

3. Compare the sets of graphs from step 1 to each other. Describe their similarities and differences, as in step 2.

4. Compare the cubic, quadratic, and quartic graphs from step 1. Which graphs are similar to the graphs of:

- $f(x) = x^2$
- $f(x) = x^3$
- $f(x) = -x^2$

Explain how they are similar.

Reflect and Respond

1. a) How are the graphs and equations of linear, cubic, and quartic functions similar?

b) How are the graphs and equations of quadratic and cubic functions similar?

2. Describe the relationship between the real behaviours of the graphs and the degree of the corresponding function.

3. What is the relationship between the sign of the leading coefficient of a function equation and the real behaviour of the graph of the function?

4. What is the relationship between the constant term in a function equation and the position of the graph of the function?

5. What is the relationship between the minimum and maximum number of x-intercepts of the graph of a function with the degree of the function?

Link the Ideas

The degree of a **polynomial function** in one variable, x , is n , the greatest power of x in the function. The coefficient of the term whose value is not affected by the variable is the constant term, a_0 . In this chapter, the coefficients a_1, a_2, \dots, a_n and the constant a_0 are restricted to integral values.

Polynomial Equations

- A polynomial equation in one variable is an equation of the form $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = 0$, where $a_n \neq 0$.
- A polynomial equation in two variables is an equation of the form $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 + b_m y^m + b_{m-1} y^{m-1} + \dots + b_1 y + b_0 = 0$, where $a_n \neq 0$ and $b_m \neq 0$.

3.4
Trigonometric Identities

Example 4

Determine Exact Trigonometric Values for Angles

Determine the exact value for each expression.

a) $\sin \frac{\pi}{12}$

b) $\tan 105^\circ$

Solution

a) Use the difference identity for sine with two special angles.

For example, because $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$, $\sin \frac{\pi}{12} = \sin \left(\frac{\pi}{4} - \frac{\pi}{6} \right)$.

$\sin \frac{\pi}{12} = \sin \left(\frac{\pi}{4} - \frac{\pi}{6} \right)$

$= \sin \frac{\pi}{4} \cos \frac{\pi}{6} - \cos \frac{\pi}{4} \sin \frac{\pi}{6}$

$= \left(\frac{\sqrt{2}}{2} \right) \left(\frac{\sqrt{3}}{2} \right) - \left(\frac{\sqrt{2}}{2} \right) \left(\frac{1}{2} \right)$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

b) Method 1: Use the difference identity for tangent.

Reverse $\tan 105^\circ$ as a difference of special angles.

Use the tangent difference identity, $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$.

$\tan(135^\circ - 30^\circ) = \frac{\tan 135^\circ - \tan 30^\circ}{1 + \tan 135^\circ \tan 30^\circ}$

$= \frac{-1 - \frac{1}{\sqrt{3}}}{1 + (-1) \left(\frac{1}{\sqrt{3}} \right)}$

$= \frac{-\frac{\sqrt{3} + 1}{\sqrt{3}}}{\frac{\sqrt{3} - 1}{\sqrt{3}}}$

$= \frac{-\sqrt{3} - 1}{\sqrt{3} - 1}$

Method 2: Use a Quotient Identity with Sine and Cosine

$\tan 105^\circ = \frac{\sin 105^\circ}{\cos 105^\circ}$

$= \frac{\sin(60^\circ + 45^\circ)}{\cos(60^\circ + 45^\circ)}$

$= \frac{\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ}{\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ}$

$= \frac{\left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right)}{\left(\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right) - \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right)}$

$= \frac{\frac{\sqrt{6} + \sqrt{2}}{4}}{\frac{\sqrt{2} - \sqrt{6}}{4}}$

$= \frac{\sqrt{6} + \sqrt{2}}{\sqrt{2} - \sqrt{6}}$

Your Turn

Use a sine or difference identity to find the exact values of

a) $\cos 105^\circ$ **b)** $\tan \frac{5\pi}{12}$

Key Ideas

You can use the sum and difference identities to simplify expressions and to determine exact trigonometric values for some angles.

Sum Identities

$\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)}$ $\tan(A - B) = \frac{\sin(A - B)}{\cos(A - B)}$

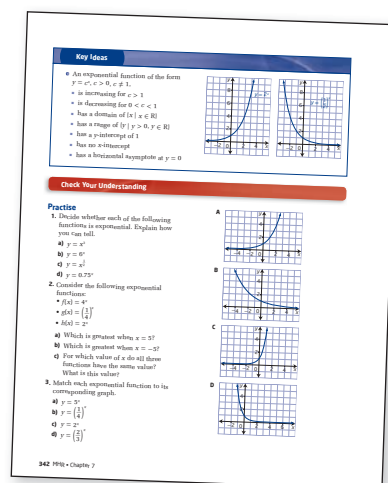
Double-Angle Identities

$\sin 2A = 2 \sin A \cos A$ $\cos 2A = \cos^2 A - \sin^2 A$ $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

$\cos 2A = 2 \cos^2 A - 1$ $\cos 2A = 1 - 2 \sin^2 A$

Check Your Understanding

- **Practise:** These questions allow you to check your understanding of the concepts. You can often do the first few questions by checking the Link the Ideas notes or by following one of the worked Examples.
- **Apply:** These questions ask you to apply what you have learned to solve problems. You can choose your own methods of solving a variety of problem types.
- **Extend:** These questions may be more challenging. Many connect to other concepts or lessons. They also allow you to choose your own methods of solving a variety of problem types.
- **Create Connections:** These questions focus your thinking on the Key Ideas and also encourage communication. Many of these questions also connect to other subject areas or other topics within mathematics.
- **Mini-Labs:** These questions provide hands-on activities that encourage you to further explore the concept you are learning.



Other Features

Key Terms are listed on the Chapter Opener pages. You may already know the meaning of some of them. If not, watch for these terms the first time they are used in the chapter. The meaning is given in the margin. Many definitions include visuals that help clarify the term.

Some **Did You Know?** boxes provide additional information about the meaning of words that are not Key Terms. Other boxes contain interesting facts related to the math you are learning.

Opportunities are provided to use a variety of **Technology** tools. You can use technology to explore patterns and relationships, test predictions, and solve problems. A technology approach is usually provided as only one of a variety of approaches and tools to be used to help you develop your understanding.

Web Links provide Internet information related to some topics. Log on to www.mcgrawhill.ca/school/learningcentres and you will be able to link to recommended Web sites.

Key Terms

logarithmic function
logarithm
common logarithm
logarithmic equation

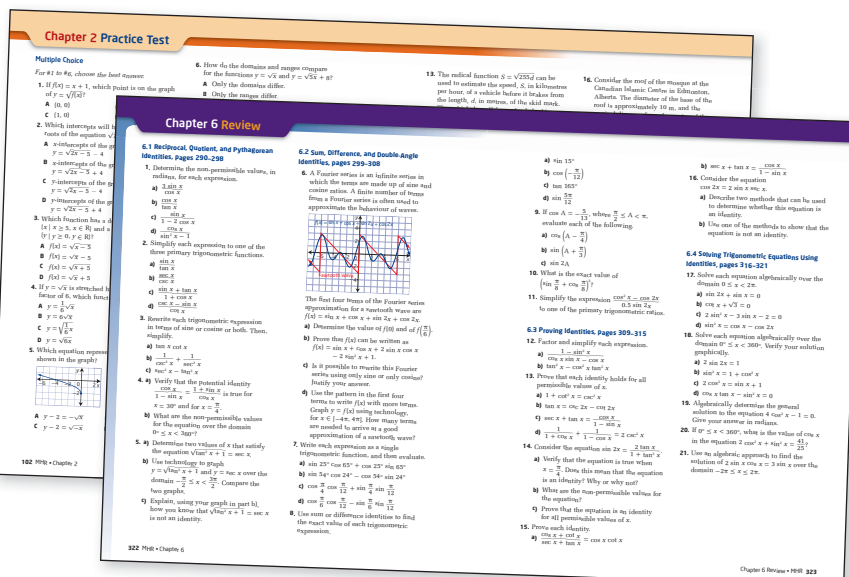
Did You Know?

The SI unit used to measure radioactivity is the becquerel (Bq), which is one particle emitted per second from a radioactive source. Commonly used multiples are kilobecquerel (kBq), for 10^3 Bq, and megabecquerel (MBq), for 10^6 Bq.

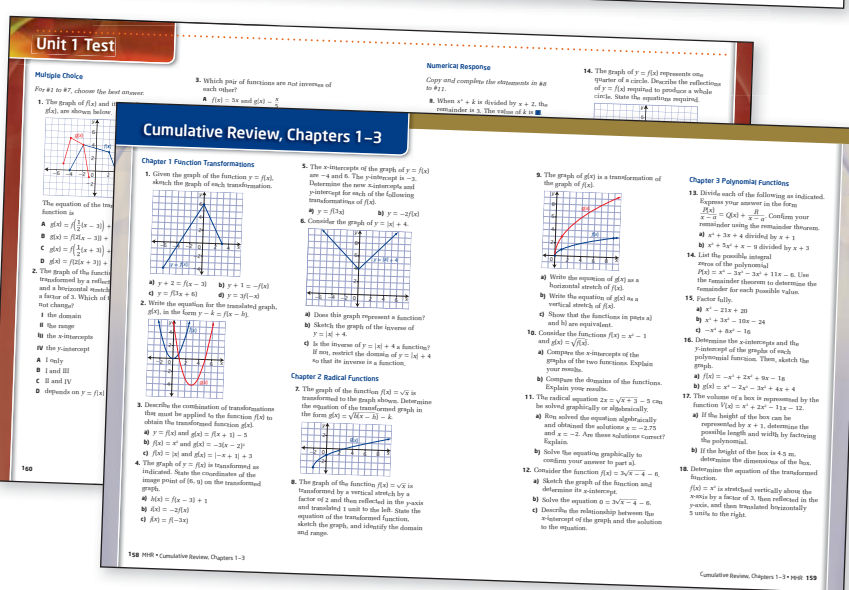
Web Link

To learn more about a career in radiology, go to www.mcgrawhill.ca/school/learningcentres and follow the links.

A **Chapter Review** and a **Practice Test** appear at the end of each chapter. The review is organized by section number so you can look back if you need help with a question. The test includes multiple choice, short answer, and extended response questions.



A **Cumulative Review** and a **Unit Test** appear at the end of each unit. The review is organized by chapter. The test includes multiple choice, numerical response, and written response questions.



Answers are provided for the Practise, Apply, Extend, Create Connections, Chapter Review, Practice Test, Cumulative Review, and Unit Test questions. Sample answers are provided for questions that have a variety of possible answers or that involve communication. If you need help with a question like this, read the sample and then try to give an alternative response.

Refer to the illustrated **Glossary** at the back of the student resource if you need to check the exact meaning of mathematical terms.

If you want to find a particular math topic in *Pre-Calculus 12*, look it up in the **Index**, which is at the back of the student resource. The index provides page references that may help you review that topic.