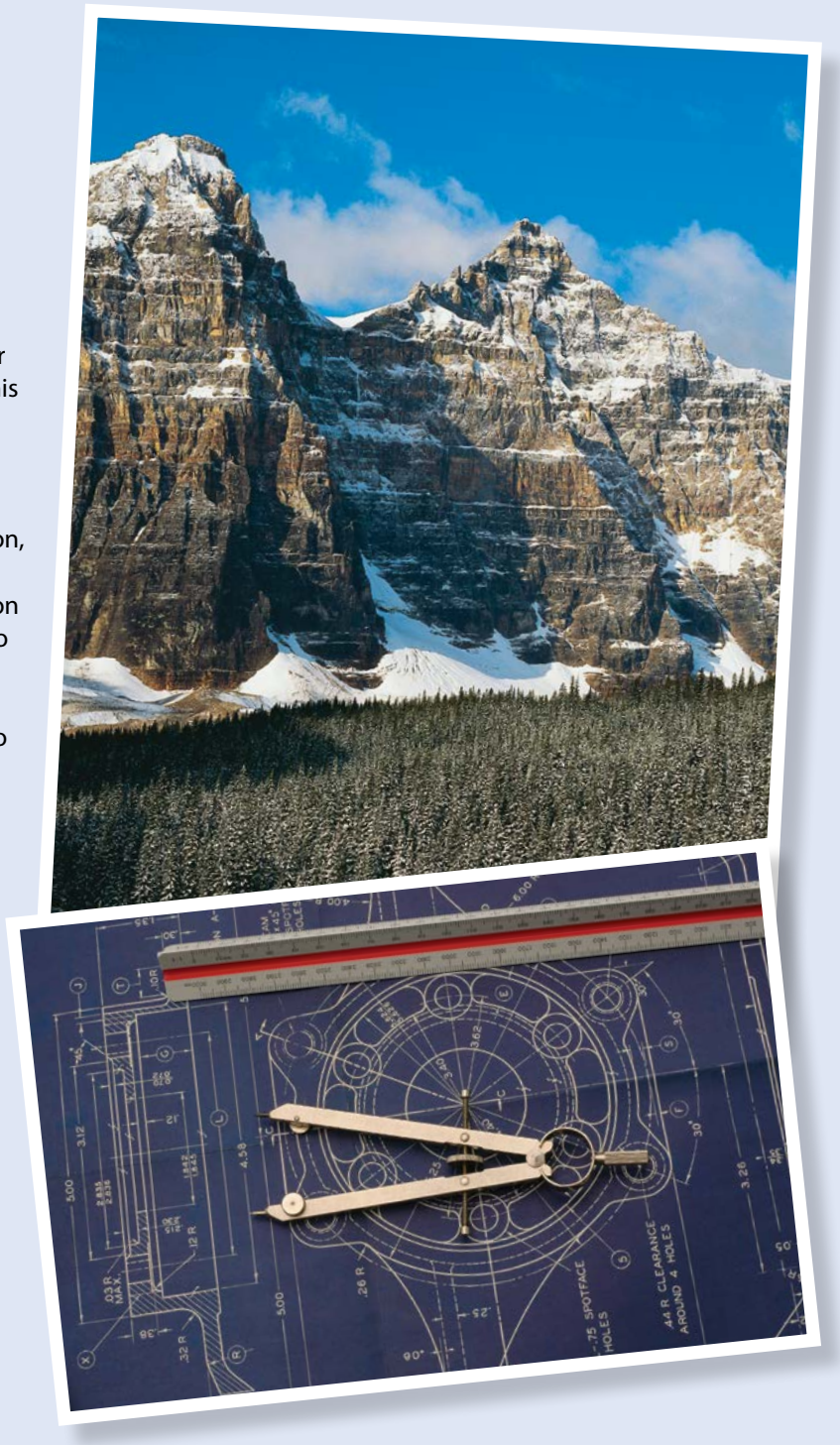
Chapter

2

Scale Factors and Similarity

Have you ever wondered how the scale on a map works? What about how tall a cliff or a mountain is, or how far it is to the other side of a river or ravine? In this chapter, you will explore the SI measurement system and scale factors. You will also use the properties of similar triangles to find heights of very tall objects.

Workers in fields such as architecture, road construction, and surveying often use the SI measurement system, scale factors, and diagrams. For example, a draftsperson prepares technical drawings and plans that are used to build everything from industrial machinery to skyscrapers. The drawings show the details of the structures and specify the dimensions, the materials to be used, and the procedures to be followed.



Big Idea

Photographs, models, and drawings are often enlargements or reductions of the original. The scale factor is the multiplicative relationship between them. You can use proportional reasoning to understand how the changes in size are related.

Inquire and Explore

- How can you describe, measure, and compare spacial relationships?
- How are similar shapes related?
- What characteristics make shapes similar?


YUKON
NWT
TN-O
ALASKA
USA
É-U d'A
BRITISH COLUMBIA
COLOMBIE-BRITANNIQUE
CANADA
N
Atlin
Atlin L
Liard R
Dease Lake
Stikine R
Fort Nelson
Stewart
Hudson's Hope
Fort St John
Williston L
Dawson Creek
New Hazelton
Chetwynd
Dixon Entrance
Prince Rupert
Mackenzie
Smithers
Terrace
Tumbler Ridge
Îles de la Reine-Charlotte
Masset
Kitimat
Houston
Fort St James
Queen Charlotte City
Burns Lake
Fraser Lake
Sandspit
Vanderhoof
Fr. Fraser R.
Coast Mountains
Prince George
Queen Charlotte Islands
Hecate Str
McBride
Quesnel
Rocky Mountains / Montagnes Rocheuses
ALBERTA
Wells
Valemount
Bella Bella
Bella Coola
Williams Lake
Kinbasket L
Queen Charlotte Sound
Chaine Côtière
Fraser R.
100 Mile House
PACIFIC OCEAN
OCÉAN PACIFIQUE
Revelstoke
Golden
Port Hardy
Port McNeill
Lillooet
Ashcroft
Port Alice
Campbell River
Kamloops
Salmon Arm
Columbia R.
Invermere
Powell River
Merritt
Vernon
Whistler
Nakusp
Elkford
Vancouver Island
Courtenay
Squamish
Okanagan L
Kelowna
Kimberley
Île de Vancouver
Port Alberni
Sechelt
Penticton
Nelson
Cranbrook
Vancouver
Hope
Castlegar
Tofino
Georgia Str
Abbotsford
Osoyoos
Trail
Creston
Ucluelet
Parksville
Nanaimo
Sidney
Ladysmith
Saanich
Victoria
UNITED STATES OF AMERICA
ÉTATS-UNIS D'AMÉRIQUE
Juan de Fuca Str
Dét de Juan de Fuca
Scale / Échelle
100 km 0 100 200 300 km
LEGEND / LÉGENDE
Provincial capital / Capitale provinciale
Other populated places / Autres lieux habités
Trans-Canada Highway / La Transcanadienne
Major road / Route principale
Ferry route / Traversier
International boundary / Frontière internationale
Provincial boundary / Limite provinciale
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Get Ready

Multiplication Patterns

1. Evaluate using mental math.

a) 14×1 **b)** 14×10

c) 14×100 **d)** 14×1000

2. Evaluate using mental math.

a) 0.355×1000 **b)** 1.89×1000

c) 0.75×100 **d)** 2.63×100

3. a) Describe a pattern when multiplying by 10, 100, or 1000.

b) Explain to a classmate how to mentally determine $346 \times 10\,000$.

c) Ask your classmate to explain how to mentally determine $3.46 \times 10\,000$.

Division Patterns

4. Evaluate using mental math.

a) $2600 \div 1$ **b)** $2600 \div 10$

c) $2600 \div 100$ **d)** $2600 \div 1000$

5. Evaluate without using a calculator.

a) $473 \div 1$ **b)** $473 \div 10$

c) $473 \div 100$ **d)** $473 \div 1000$

6. a) Describe a pattern when dividing by 10, 100, or 1000.

b) Explain to a classmate how to mentally determine $700\,000 \div 100\,000$.

c) Ask your classmate to explain how to mentally determine $750\,000 \div 100\,000$.

Proportions

A proportion is a relationship that says that two ratios are equivalent. You can write proportions in fraction form.

$$\overset{\times 3}{\frac{2 \text{ km}}{3 \text{ h}}} = \underset{\times 3}{\frac{6 \text{ km}}{9 \text{ h}}}$$

To solve for an unknown quantity in a proportion, isolate the unknown quantity.

$$\frac{1}{10} = \frac{x}{24}$$

$$\frac{1}{10}(24) = \frac{x}{24}(24) \quad \text{Multiply both sides by 24.}$$

$$\frac{24}{10} = x$$

7. Solve each equivalent fraction. Do as many as you can using mental math.

a) $\frac{4}{5} = \frac{\blacksquare}{15}$

b) $\frac{2}{3} = \frac{\blacksquare}{12}$

c) $\frac{8}{12} = \frac{\blacksquare}{6}$

d) $\frac{\blacksquare}{14} = \frac{1}{7}$

8. Solve for each unknown quantity. Do as many as you can using mental math.

a) $\frac{6}{5} = \frac{18}{x}$ **b)** $\frac{9}{x} = \frac{36}{24}$

c) $\frac{16}{22} = \frac{32}{x}$ **d)** $\frac{5.5}{x} = \frac{11}{3.6}$

9. Solve for each unknown quantity.

a) $\frac{1}{10} = \frac{x}{24}$ **b)** $\frac{1}{5} = \frac{x}{12}$

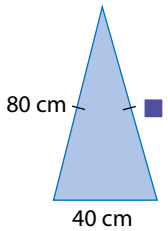
c) $\frac{2}{3} = \frac{x}{16}$ **d)** $\frac{x}{12} = \frac{4}{30}$

Working With Diagrams

Hash marks on a diagram identify sides that are the same length. Hash marks look like this: / or //.

10. Solve for the unknown values.

a)



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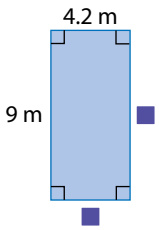

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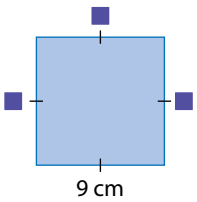
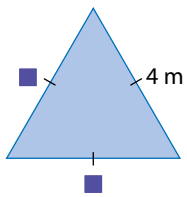


d)

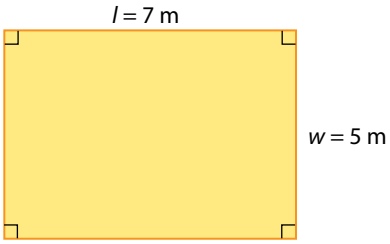


11. a) In #10a), the angle at the top of the triangle is 30°. Determine the measure of the other two angles inside the triangle. Explain your reasoning.

b) In #10b), why do you think the diagram does not need hash marks?

c) In #10d), what is the measure of each interior angle? Explain your reasoning.

12. a) What is the perimeter of the rectangle?



b) What is the area of the rectangle?

c) If the width of the rectangle is doubled, what do you think happens to the perimeter?

d) If the width of the rectangle is doubled, what do you think happens to the area?

e) Calculate the perimeter and the area if the width of the rectangle is doubled. Check your answers against your predictions in parts c) and d).

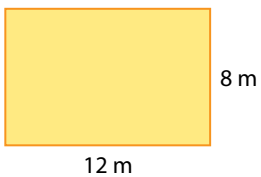
Scale

A scale shows the relationship between a distance on a drawing, model, or map and the actual distance.

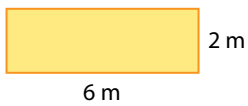
A scale of 1 cm : 1 m means that 1 cm on the diagram represents 1 m actual size.

13. Draw a scale diagram of each rectangle. Use a scale of 1 cm represents 2 m.

a)



b)



14. A road map uses a scale of 1 cm : 7 km. What is the actual distance between 2 towns that are 6 cm apart on the map?

2.1

SI Measurement

Focus On ...

In this lesson, I will learn to

- solve problems using a personal referent to estimate measurements
- convert between SI measurements

Explore and Analyze

Rob Cardinal is a researcher at the University of Calgary's Rothney Astrophysical Observatory. He is a member of the Siksika First Nation. While looking for an asteroid, he ended up taking images of an undiscovered comet. The comet was named Comet Cardinal. Whether exploring outer space or travelling the world, we often use linear (length or distance) measurements with SI units to measure distances. SI stands for Système International d'Unités. Estimating lengths or distances is easier if you can approximate common measurements using your body or your surroundings. How can you use personal references to estimate SI measurements?

referent

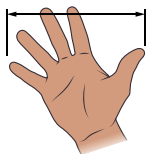
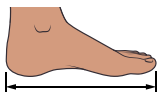
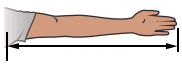
- something familiar or convenient that is used to refer to something else
- you can also use the word *reference*

1. Hold a metre stick at your side with one end of the stick touching the floor. How high up your body does the other end reach? You can use this location on your body as a personal **referent** for 1 metre. The referent does not need to match the measurement exactly, but it needs to be pretty close.

2. Copy and complete the table as you collect personal referents for common SI lengths.

SI Measure	Personal Referent
1 m	
10 cm	
1 cm	
1 mm	

3. Use your body to create other personal referents for SI lengths. Copy and complete the table.

Personal Referent		SI Measure
Your outstreched hand		
The length of your foot		
The length of your arm		
Your height		

The length, width, or height of some things is often standard. In most homes, all of the doors are the same height. Again, you can use a referent to estimate, even if the measurement is not exact. For example, you can use the width of a standard piece of paper as a reference for 20 cm even though it is a little wider than 20 cm.

4. Copy and complete the table.

Referent	SI Measure	Referent for Approximate SI Measure
Width of a piece of paper	21.5 cm	20 cm
Height of a door		
Height of a desk or table		
Height of an electrical outlet		
Length of a piece of paper		

5. You can use your collection of measurement referents to estimate different heights. For example, a student might be 5.5 paper lengths tall. Estimate the height of your classroom.

Develop Understanding

Example 1: Estimate and Measure Using SI Units

Use a referent to estimate each distance. Then, measure each distance.

a) the thickness of a cellphone

b) the height of the seat of a chair

c) the width of the cover of this textbook

Solution

a) Use a referent: The width of one fingernail is about 1 cm.

The thickness of a cellphone is approximately as wide as half of one fingernail. An estimate of the thickness of a cellphone is 0.5 cm.

Measure using a ruler: The thickness of a cellphone is 4.5 mm or 0.45 cm.

b) Use a referent: For most people, 1 metre is about waist height.

The height of the seat of a chair is approximately half of the height of 1 metre, so a reasonable estimate is 50 cm.

Measure using a measuring tape: The height of the seat of a chair is 46 cm.

c) Use a referent: Lay a standard sheet of paper over the cover of the book. The width of the paper is a reference for 20 cm. The book is also about 20 cm wide.

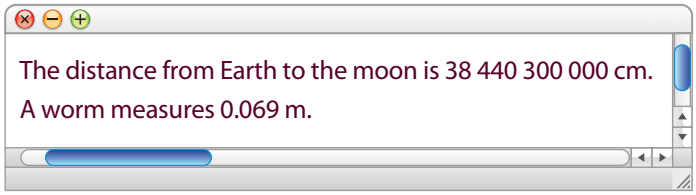
Measure using an SI ruler: The width of the cover of this book is 22.8 cm.

Show You Know

Use a reference to estimate the height of the chalk- or marker-tray on a blackboard or whiteboard. Then, measure the height.

Example 2: Convert Between SI Units for Length

A website reports the following measurements in different stories.



a) State a more appropriate SI unit for each measurement. Justify your choice.

b) Convert the given measurement to the more appropriate unit.

Solution

a) Earth is very far from the moon. You could measure the distance in kilometres rather than centimetres. The length of a worm is very small. You could measure it in millimetres or centimetres rather than metres.

b) Convert 38 440 300 000 cm to kilometres.

Use *unit analysis* to determine the number of centimetres in 1 km.
1 m = 100 cm and 1 km = 1000 m
So, 1 km = (100 × 1000) cm
1 km = 100 000 cm

$$38\ 440\ 300\ 000\ \text{cm}\left(\frac{1\ \text{km}}{100\ 000\ \text{cm}}\right) = 384\ 403\ \text{km}$$

Represent 38 440 300 000 cm more appropriately as 384 403 km.

Convert 0.069 m to centimetres.
Let x represent the number of centimetres.
Use proportional reasoning.

$$100\ \text{cm} = 1\ \text{m}$$
$$\frac{100\ \text{cm}}{1\ \text{m}} = \frac{x\ \text{cm}}{0.069\ \text{m}}$$
$$100(0.069) = x$$
$$6.9 = x$$

It is OK if the ratios are inverted. Just make sure to match up centimetres with centimetres and metres with metres.

Convert the measurement 0.069 m to a more appropriate measure of 6.9 cm or, since 1 cm = 10 mm, 69 mm.

Show You Know

Convert each measurement to a more appropriate SI unit. Justify your choice.

a) A tube of toothpaste is 205 mm long.

b) The circumference of a highlighter measures 0.06 m.

c) You travel 418 000 m from Penticton to Vancouver.

d) The top of a door is 2032 mm high.

Connect and Reflect

Key Ideas

- Measurement referents allow you to estimate or approximate lengths or distances.
- You can use unit analysis to convert from one SI unit to another.

Practise

For help with #1 and #2, refer to Example 1 on page 42.

1. Use your collection of SI measurement references to estimate each measure in your classroom. Justify your choice of unit.

a) the height of a light switch from the floor

b) the width of your classroom

c) the length of your desk or table

2. Measure each distance in #1 and compare the measurement to your estimate.

For help with #3 to #5, refer to Example 2 on page 43.

3. State an appropriate SI unit for each measurement.

a) the diameter of a quarter

b) the length of a car

c) the thickness of a quarter

d) the diameter of Earth

4. Convert each measurement.

a) 6 cm = ■ mm **b)** 4 m = ■ cm

c) 7 km = ■ m **d)** 0.5 cm = ■ mm

e) 0.5 m = ■ cm **f)** 500 m = ■ km

5. Copy and complete the table. Convert each given length. The first one is done for you.

	Centimetres	Millimetres	cm and mm
	4.1 cm	41 mm	4 cm 1 mm
a)			3 cm 7 mm
b)		86 mm	
c)			10 cm 5 mm
d)	14.4 cm		

Apply

6. Convert each measurement to a more appropriate unit. State why you think this is a better unit to use.

a) Mount Logan, in southwestern Yukon, is 595 900 cm tall.

b) The diameter of an apple is 0.064 m.

c) The largest brown bear, the Kodiak, is 2440 mm in length.

7. Distances in most international sports are stated in SI units. The table shows the actual distances for some events. Copy and complete the table.

	Event	Metres	Kilometres
a)	Speed skating	1500 m	
b)	Olympic walking		20 km
c)	Kayaking	500 m	
d)	Cross-country running	12 000 m	
e)	Biathlon		7.5 km

8. a) Draw a letter S on a plain piece of paper using the following estimates of the curve length.

i) 30 mm **ii)** 25 cm

b) Explain how you will measure the curve length of each letter you drew.

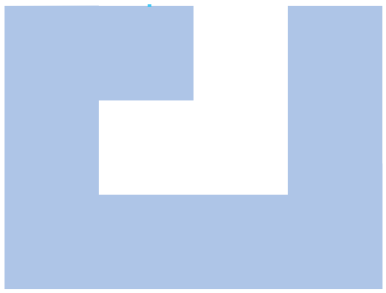
c) Measure each S and compare the measurements to the distances. If you are out by more than 5 mm for part i) or 2 cm for part ii), draw the letter again.

9. ✓ **Competency Check** When you convert between different metric units, the digits remain the same. Only the decimal point appears in a different location. Explain why this happens.

10. a) Estimate the perimeter of each figure. Use an appropriate SI unit.

i)

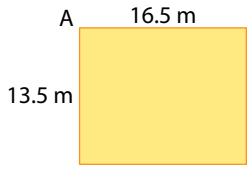
ii)



b) Determine each perimeter. Compare the measurement to your estimate. How close was your estimate?

Extend

11. A backyard has the dimensions shown. A lawn mower cuts a 52-cm width of grass.



a) You start at A and return to A when finished. What minimum distance do you walk?

b) Does your route, such as following the perimeter or walking in rows, affect the distance you walk? Explain.

12. Work with a partner. Suppose you sprained your ankle and need to wrap it with a compression bandage.

a) Estimate the length of bandage you need to wrap your foot and ankle four times.

b) Wrap your ankle and foot with a piece of cloth using a figure eight technique. Measure the length of cloth you needed. Compare your estimate with the actual measurement.

c) Suppose you estimate the length of bandage you need by estimating the circumference of your foot and ankle and multiplying the sum by four. Do you think your estimate will be close to the actual measurement of the bandage? Why or why not?

2.2

Enlargements and Reductions

Focus On ...

In this lesson, I will learn to

- analyze to determine the scale factor by which an item was reduced or enlarged
- solve for actual measurements using a given scale

Explore and Analyze

Most maps include a **scale**. The scale shows the ratio of distance on the screen or diagram and the actual distance on Earth. The map's scale in the left corner looks like a ruler. Another way of expressing scale is to use a numerical ratio of diagram measurement to actual measurement. How can you use a scale when measuring items?

scale

- the relationship between a distance on a drawing, model, or map and the actual distance
- for example, a scale of 1 cm : 1 m means that 1 cm on the diagram represents 1 m actual size

scale reduction

- the multiplier used to reduce the size of an object

scale factor

- the number used as a multiplier in scaling
- for example, to enlarge a square by a scale factor of 3, multiply each side of the original square by 3

1. A **scale reduction** is often used to illustrate large things that are not practical to draw to actual size because they are too big. Research the dimensions of a standard stop sign in Canada.

a) Measure the width of the stop sign on the photo.

b) How many times as wide is an actual stop sign as the stop sign in the photo? This is called the **scale factor**.

c) Express the scale of the photo as a *diagram measurement* : *actual measurement* ratio.

d) Express the ratio in lowest terms. Do not include the units in the ratio.

2. You can use a **scale enlargement** to show small things that are not practical to draw to actual size because they are too tiny. Research the diameter of a Canadian dime.

a) Measure the width of the dime on the photo.

b) How many times as wide is the dime in the photo as an actual dime?

c) Express the scale of the photo in the same order as you did for the stop sign, that is, as a *diagram measurement* : *actual measurement* ratio.

d) Express the ratio in lowest terms. Do not include the units in the ratio.

3. Compare the ratios in #1 and #2. What do you notice about the way the scale is expressed and the type of representation of the actual object?

scale enlargement
- the multiplier used to enlarge the size of an object

Develop Understanding

Example 1: Determine the Scale of a Photo or Diagram

The Sopwith Camel was a British World War I fighter plane. The photo shows a scale model of the Sopwith Camel. In a scale model, every dimension of the actual object has been reduced by the same factor.

The actual wingspan of the Sopwith Camel is 8.53 metres. Measure the wingspan on the photo and determine the scale of the photo. Express the scale factor as a fraction.

Solution

Measure the wingspan on the photo: 11.8 cm

The wingspan of the actual plane is 8.53 m, which is 853 cm.
$853 \div 11.8 = 72.2881\ldots$

Why do you need to convert 8.53 m to 853 cm?

The photo is approximately a 1 : 72 reduction of the size of the actual Sopwith Camel. Expressed as a fraction, the actual Sopwith Camel has been reduced by a scale factor of $\frac{1}{72}$ for the photo.

Show You Know

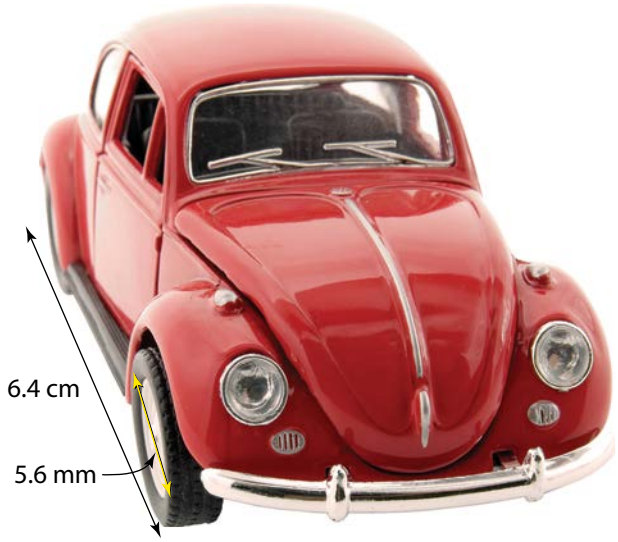
Determine the scale of this image of a grey whale. The average grey whale is about 14 m in length. Express the scale as a ratio and as a fraction.

Example 2: Determine the Actual Size of an Object

A common scale for collectible toy cars is 1 : 64.

a) What does the ratio mean?

b) Use the measurements on the photo of the scale model to determine the actual measurements of the 1959 Volkswagen Beetle. Express your answer in the most appropriate SI units. Round your answers as necessary.



Solution

a) The 1 : 64 ratio means that each of the actual measurements of the real car is 64 times the length of the same measurement on the model.

b) Apply the scale to the car length measurement on the model.
$6.4 \times 64 = 409.6$

The overall length of an actual 1959 Volkswagen Beetle is 409.6 cm.
Rounded to the nearest cm, the overall length of the car is 410 cm.

Convert 410 cm to metres.
Let x represent the number of metres.
Use proportional reasoning.

$$100 \text{ cm} = 1 \text{ m}$$
$$\frac{1 \text{ m}}{100 \text{ cm}} = \frac{x \text{ m}}{410 \text{ cm}}$$
$$\frac{410}{100} = x$$
$$4.1 = x$$

The Beetle is 4.1 m long.

Apply the scale to the wheel diameter measurement on the model.
$5.6 \times 64 = 358.4$

The actual wheel diameter of the 1959 Volkswagen Beetle is 358.4 mm.
You can also express this length as 35.84 cm, or, as a rounded value, 36 cm.

Show You Know

This photo of a humpback whale is a $\frac{1}{125}$ reduction. How long is the whale's pectoral (top) fin?

Connect and Reflect

Key Ideas

- Scale is a ratio of the measurements of any model (such as a map, drawing, or photo) to the measurements of the actual item.
- If the smaller of the numbers of the scale is first, it is a reduction.
- If the larger of the numbers of the scale is first, it is an enlargement.

Practise

For help with #1 and #2, refer to Example 1 on page 47.

1. State whether each scale is a scale reduction or an enlargement. Explain how you know.

a) 27 : 1 **b)** 1 : 27

c) 1 : 1.5 **d)** 10 000 : 1

2. Determine the scale in each case and state whether the scale indicates a reduction or an enlargement.

	Measurement on Model	Actual Measurement
a)	0.5 cm	1 m
b)	720 mm	36 cm
c)	1 cm	0.1 mm
d)	5 cm	5 cm

For help with #3 and #4, refer to Example 2 on page 48.

3. Calculate the actual length given the scale and the length on a scale model.

a) 2 cm on a 1 : 72 model

b) 1.5 cm on a 1 : 24 reduction

c) 1 cm on a 100 : 1 enlargement

d) 1 mm on a 25 : 1 enlargement

4. Calculate the actual size given the scale and the length on a scale model. Give your answers in the stated units.

a) 1.5 cm on a 1 : 150 000 map, in metres

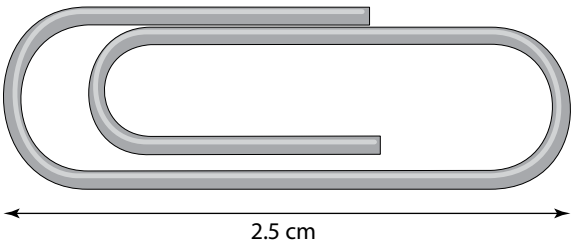
b) 3.6 cm on a 1 : 28 scale model, in millimetres

c) 0.45 mm on a 50 000 : 1 microscope enlargement, in millimetres

d) 25 cm on a 10 : 1 photo enlargement, in millimetres

Apply

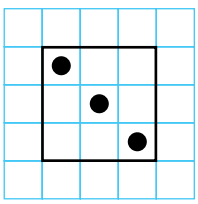
5. A paper clip is 2.5 cm long. Determine the scale of this drawing of the paper clip.



6. a) Measure the length and the width of this book, to the nearest centimetre. On centimetre grid paper, make a 1 : 2 scale reduction of the front cover.

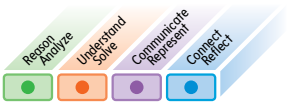
b) On the same paper, make a 1 : 4 reduction.

7. The sketch shows the number three on a die. Create a 3 : 1 enlargement.



8. Research the average length of a housefly. Determine the scale of the photo.

9. ✓ **Competency Check**



a) Estimate the scale of this picture of a bowl. Is it an enlargement or a reduction?

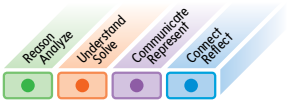
b) Explain what a 1 : 1 scale means.

c) Creative advertisers can enlarge the size of items so that they look bigger or reduce them so they look smaller. The fruit in this picture have been reduced by a factor of 1.5. Why might an advertiser choose to enlarge or reduce the size of some items?

10. The *All My Relations* mural in Allan Gardens, Toronto, was installed in 2012 to beautify temporary fencing around a construction site. The mural gave the Aboriginal people the opportunity to tell their story in four sections dedicated to community, water, traditional teachings, and the history of the land. The entire mural was about 274 metres long. The photo shows one section. Determine the scale of the photo.

Extend

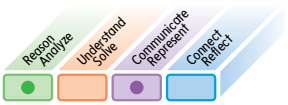
11. ✓ **Competency Check** Measurements in the sport of curling are in feet, not metres. The diameter of the rings is 4 feet, 8 feet, and 12 feet.



a) Make a scale reduction of the curling rings. Indicate the scale on your drawing.

b) Research the conversion factor between centimetres and feet. Convert the diameter of the curling rings to SI units.

12. ✓ **Competency Check** Research the dimensions of the standard Canadian flag. Research the ratio of the red : white : red bands. Draw a scale reduction of the flag. Keep the ratios of the flag intact. Indicate the scale on your drawing. Note: The maple leaf only needs to be approximately to scale, but the rest of your diagram should be very accurate.



a) Discuss the width to height ratio of the flag.

b) Discuss the red : white : red band width ratio of the flag.

2.3

Scale Diagrams

Focus On ...

In this lesson, I will learn to

- represent a place or object by creating a scale diagram
- use scale diagrams to determine unknown measurements

Explore and Analyze

Architects, surveyors, interior designers, and carpenters all make and use scale drawings in their work. Scale diagrams help you visualize actual objects. You can measure any part of a scale diagram and use the scale factor to determine measurements of the actual object. Look around your classroom. What do you see that could have started from a scale drawing?

1. Measure, to the nearest 0.5 m, the length and the width of your classroom. If the room is not rectangular, take the most important measurements.

scale diagram

- a 2-D drawing that represents a place or object
- uses scale to show the relationship between the distances on the diagram and the corresponding distances in real life

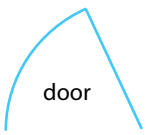
2. Choose an appropriate scale and create a **scale diagram** that represents the top view of your classroom. Pick a scale so that the diagram will take up most of the space on the paper.

3. a) Measure the distance from the door of your classroom to the nearest corner. Measure the width of the door opening.

b) Erase a "hole" at the appropriate location of your diagram for the door opening.

c) Which side of the opening are the hinges on? Does the door open into the classroom or out?

d) Draw the symbol to show the door. The straight line comes out from the hinges, and the curve represents the path of the door as it swings open.



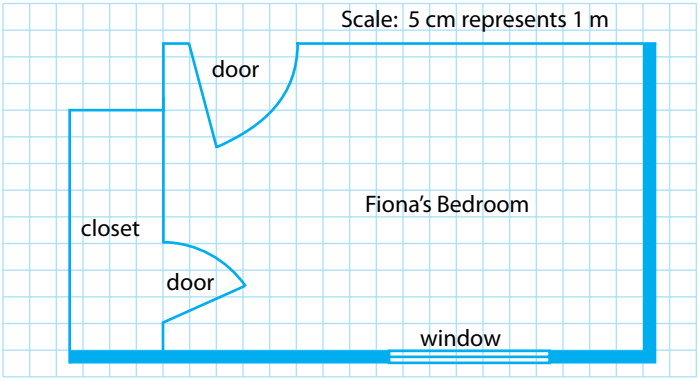
4. a) Write the scale of your diagram, as a 1 : ■ ratio, in the bottom corner of your drawing.

b) Measure one of the diagonals on your diagram. Use the scale of your drawing to determine the actual distance in your classroom. Measure the actual distance and compare the results.

Develop Understanding

Example 1: Read Scale Diagrams

This scale drawing of Fiona's bedroom is on centimetre grid paper. Her bedroom is in one corner of the house. Exterior walls are shown as half a square thick. Interior walls, which are much thinner, are shown as lines.



a) Explain the scale of the diagram.
b) Convert the scale of the diagram to a 1 : ■ ratio.
c) What are the dimensions of Fiona's bedroom? Explain how you know.
d) How wide is each door?
e) How deep is the closet?

Solution

a) The scale means that the side length of 5 squares on the diagram represents 1 metre of distance in the actual bedroom.

b) In the diagram, 5 cm (squares) = 1 metre, which equals 100 cm. The side of 1 square on the diagram represents 20 cm in the bedroom. Since the diagram is on centimetre grid paper, it is a 1 : 20 reduction.

c) On the diagram, the dimensions are 18 squares by 11.5 squares. To determine actual measures, multiply each measurement by 20. The actual dimensions are 360 cm (3.6 m) by 230 cm (2.3 m).

d) The door to the room is 4 squares wide, so it measures 80 cm.
The door to the closet is 3 squares wide, so it measures 60 cm.

e) The closet is $3\frac{1}{2}$ squares deep, so the depth of Fiona's closet is 70 cm.

Show You Know

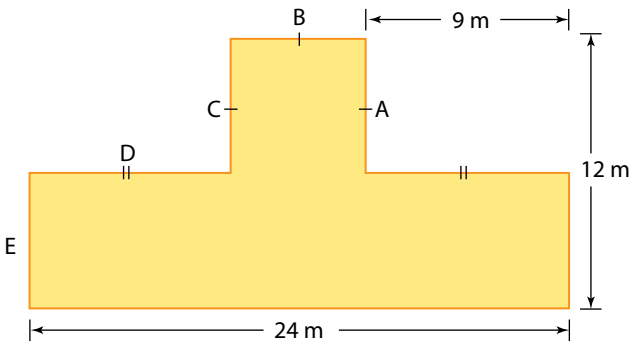
Refer back to the scale diagram of the bedroom.

a) How wide is the actual window?

b) How wide is Fiona's closet?

Example 2: Determine Missing Dimensions

To avoid clutter, many scale drawings include a minimum amount of information. Use the measurements given to determine the lengths of walls A, B, C, D, and E.



Solution

Wall D:

D is the same length as the wall with the same number of hash marks. So, wall D is 9 metres long.

Wall B:

The length of the walls parallel to the bottom wall add up to 24 metres. You know one wall is 9 metres and Wall D is also 9 metres. $9 + 9 = 18$

Therefore, B is $24 - 18 = 6$ metres long.

Walls A and C:

The hash marks show that walls A and C are the same length as B, so they are also 6 metres long.

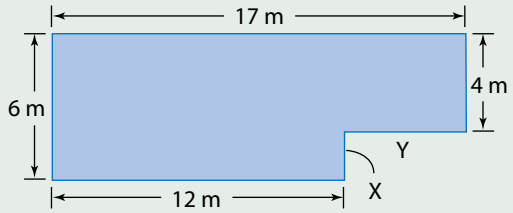
Wall E:

Since A is 6 metres and the overall dimension on the left of the diagram is 12 metres, E must also be 6 metres long.

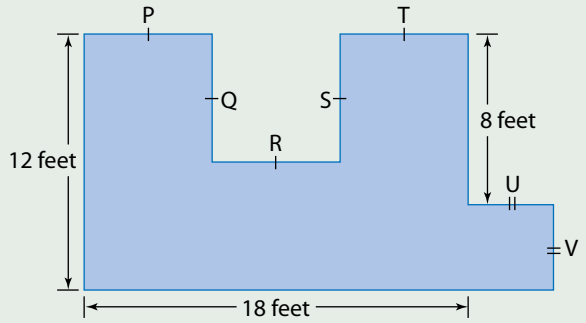
Show You Know

Determine the missing dimensions.

a)



b)



Connect and Reflect

Key Ideas

- A scale diagram helps you visualize an actual object.
- You can measure any part of a scale diagram and then use the scale factor to determine the measurements of the actual object.
- You can use mathematical reasoning to determine measurements that have not been included on a scale diagram.

Practise

For help with #1 to #3, refer to Example 1 on page 53.

1. Express each scale in ratio form.

a) 1 cm to 1 m

b) 1 cm to 0.5 m

c) 1 cm to 1 km

d) 1 cm to 100 m

2. Determine the actual distance represented by the following lengths on a scale diagram using a 1 : 50 scale.

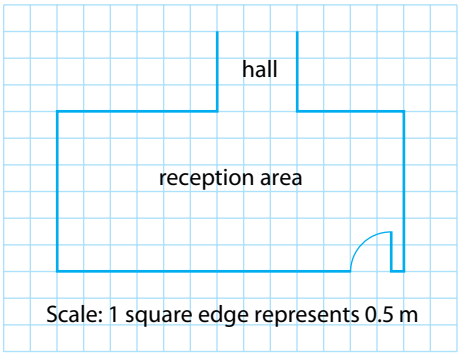
a) 1.5 cm

b) 4 cm

c) 7 cm

d) 12.5 cm

3. A dentist is hiring a local renovation company to update the reception area of the office.



a) How many square edges represent 1 metre on the drawing? How do you know?

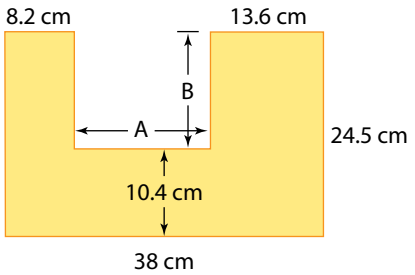
b) Count the squares on the drawing to determine the dimensions of the reception area.

c) Determine the area of the floor of the reception area. Include the area where the door swings open.

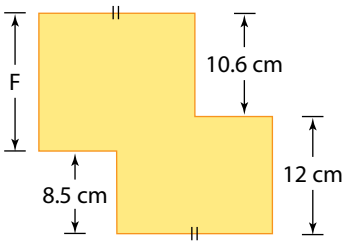
For help with #4, refer to Example 2 on page 54.

4. Determine the unknown dimensions.

a)

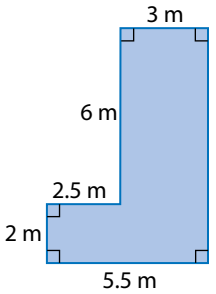


b)



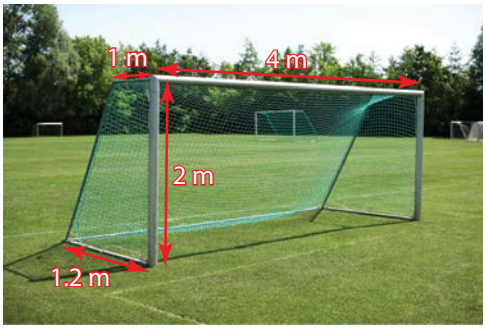
Apply

5. The drawing shows the layout for kitchen flooring. Using grid paper, create a scale diagram. Use an appropriate scale.



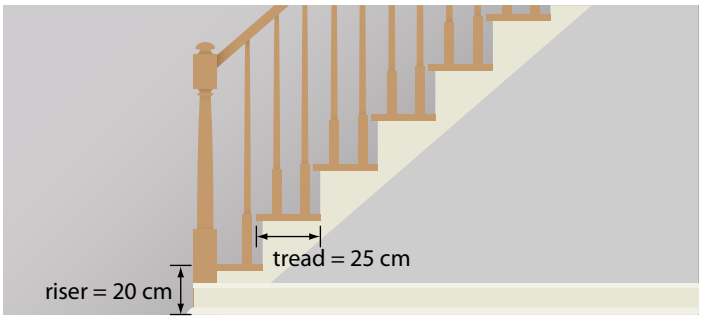
6. a) Use a scale of 1 square edge to 10 cm to create a scale drawing of the front of the soccer goal shown.

b) Use the same scale to draw the side of the goal.

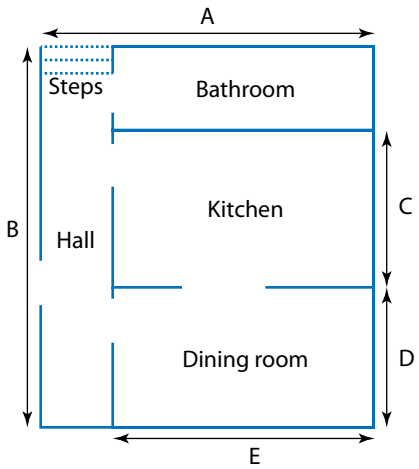


7. a) The diagram below shows part of a set of 12 stairs up to a landing. Determine the overall height and length of the stairs from the floor to the landing.

b) Without measuring and using the scale of the diagram, how can you determine how long the bannister needs to be?



8. This floor plan shows part of the ground floor of a cottage. The scale of the diagram is 1 : 100.

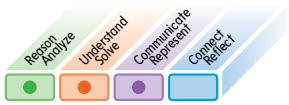


a) Determine the actual dimensions of the kitchen and the bathroom.

b) If you walked from the steps to the centre of the dining room, approximately how far would you walk? Explain your reasoning.

9. Meghan received a Pacific Northwest eagle pendant for her birthday. It measures 3.5 cm in diameter. Meghan would like to have some earrings made to match her pendant. What scale should she ask the jeweller to use when making her earrings?

10. **Competency Check** You are designing your closet to organize your belongings.



a) The closet is 1.5 m wide and 3 m high. Choose an appropriate scale and draw a scale diagram of the closet as if you were looking into it.

b) Design the interior of the closet with the following features:

- at least 1 rod for hanging pants
- at least 1 rod for hanging shirts
- drawers or baskets for socks, underwear, and other small items
- compartments or a rack for sports equipment
- shelves for sweaters, books, and other similar items

c) Add one other item of your choice to your closet design.

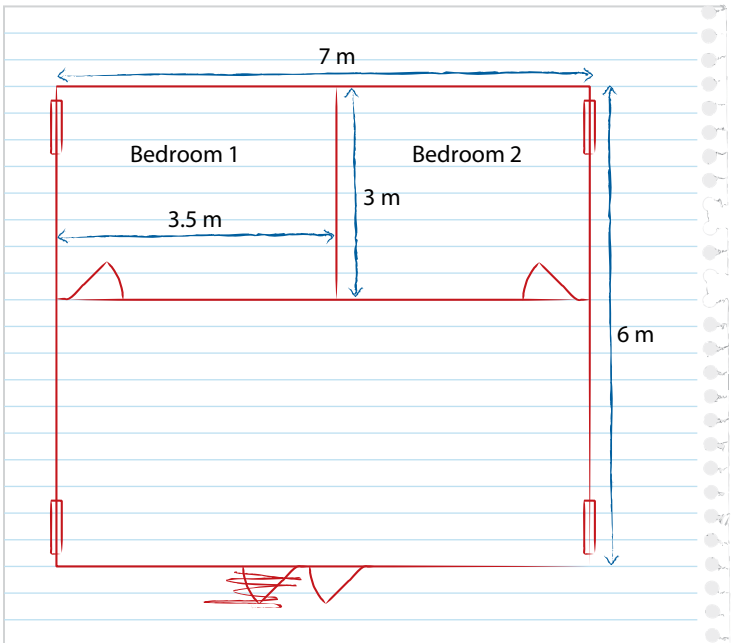
11. Tracy took a picture of a wind turbine at the wind farm in Cowley Ridge, Alberta. The height of the turbine is 45 m.

a) What scale factor was used to make this reduction?

b) What is the length of a wind turbine blade?

Extend

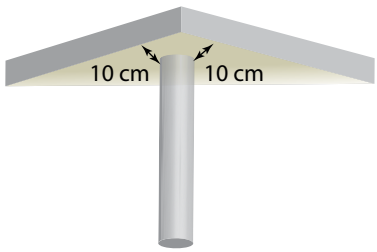
12. Bryce is renovating his family's cabin. To help plan, he makes a quick sketch of the layout. The diagram is not to scale.



a) The exterior walls and the wall between the bedrooms are made of log and are 20 cm thick. Determine the missing dimension of bedroom 2.

b) Draw a scale diagram using the scale 1 square : 20 cm. The bedroom doors are 80 cm wide and 15 cm from the corner of the room. The main door is 1 m wide and is centred on the wall. Each window is 110 cm wide and starts 50 cm from closest corner.

13. The legs of a table have a diameter of 10 cm. The legs are attached 10 cm from the edges of the tabletop, as shown in the diagram. The tabletop is 5 cm thick. The table is 150 cm long, 90 cm wide, and 75 cm high.



a) Draw a scale diagram of the front view and a side view of the table.

b) Describe what the top view would look like.

c) Create a scale diagram of the view of the table as it would look from the bottom.

14. Locate all the light switches and all the electrical outlets in your classroom. Research the symbols that are commonly used to indicate these items on diagrams. Add them to your scale diagram of the classroom that you made in the Explore and Analyze section. Add any other items that you believe would be helpful to someone who might want to renovate the room.

Note: Since your diagram is a "top view" of your classroom, do not indicate the height of any of these items, just their horizontal position along the wall.

2.4

Similar Triangles

Focus On ...

In this lesson, I will learn to

- analyze to determine whether two triangles are similar
- use similarity between triangles to solve problems

Explore and Analyze

One of the world's tallest totem poles is in Alert Bay, British Columbia. It stands approximately 53 metres tall! You can use shadows and proportional reasoning to determine the height of objects. How could you use the length of the shadow of a totem pole and of an object of known height to determine the height of the pole?

1. Work in a group. Measure the length of the shadow of your school's flagpole or a very tall tree.
2. Hold a metre stick at a right angle to the ground. Measure the length of the shadow cast by the metre stick.
3. Use the shadow lengths and proportional reasoning to determine the height of the flagpole.
4. **a)** Make a scale diagram of the items and their shadows.
   b) Draw lines from the top of each item to the end of each shadow to create two triangles.
   c) How do the angles inside one of the triangles compare to the angles inside the other?

If drawn accurately, your diagram consists of two **similar triangles**. For triangles to be similar, their **corresponding angles** must be equal and their **corresponding sides** must be proportional.

similar triangles

- triangles with equal corresponding angles and proportional corresponding sides
- $\triangle ABC \sim \triangle DEF$ means that triangle ABC is similar to triangle DEF

corresponding angles

- have the same relative position in a pair of similar triangles
- are equal

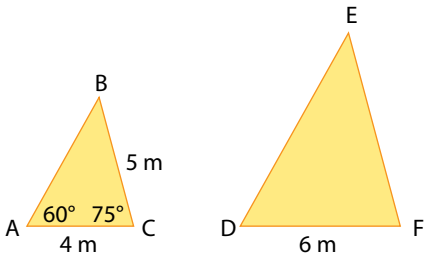
corresponding sides

- have the same relative position in a pair of similar triangles

Develop Understanding

Example 1: Determine Unknown Angle and Side Measures

Given that $\triangle ABC \sim \triangle DEF$, determine the measure of $\angle E$ and the length of EF.



Solution

Find the measure of the unknown angle, $\angle B$.

$\angle B = 180° - 60° - 75°$

$= 45°$

Similar triangles have equal corresponding angles.

Given that $\triangle ABC \sim \triangle DEF$, then $\angle E = \angle B = 45°$.

Next, determine the length of the unknown side.

Method 1: Use Proportional Reasoning Within a Triangle

BC = 5 m and AC = 4 m

$\frac{BC}{AC} = \frac{5}{4} = 1.25$

BC is 1.25 times the length of AC.

Similar triangles have proportional corresponding sides.

Given that $\triangle ABC \sim \triangle DEF$, then EF must be 1.25 times the length of DF.

$6 \times 1.25 = 7.5$

EF is 7.5 metres long.

Method 2: Use Proportional Reasoning Between Triangles

Sides AC and DF are corresponding sides, which means they are proportional.

$\frac{DF}{AC} = \frac{6}{4} = 1.5$

Since the triangles are similar, the dimensions of $\triangle DEF$ are 1.5 times those of $\triangle ABC$.

Sides BC and EF are corresponding sides, which means they are proportional.

$EF = 1.5 \times BC$

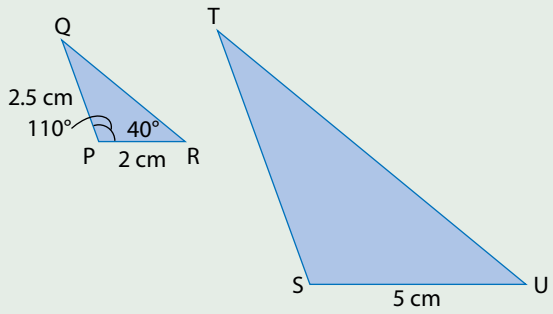
$= 1.5 \times 5$

$= 7.5$

EF is 7.5 metres long.

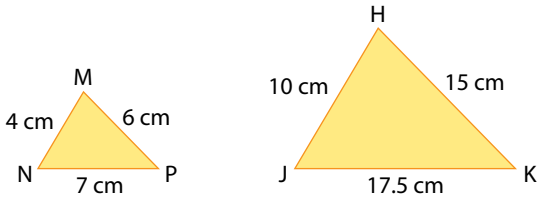
Show You Know

Given that △PQR ~ △STU, determine the measure of ∠T and the length of ST.



Example 2: Determine Whether Two Triangles Are Similar

Show that △MNP ~ △HJK.



The side opposite ∠M is labelled *m*; the side opposite ∠P is *p*; and so on.

Solution

Two triangles are similar if their corresponding angles are equal or if their corresponding sides are proportional.

Write ratios comparing the longest sides, the shortest sides, and the third sides.

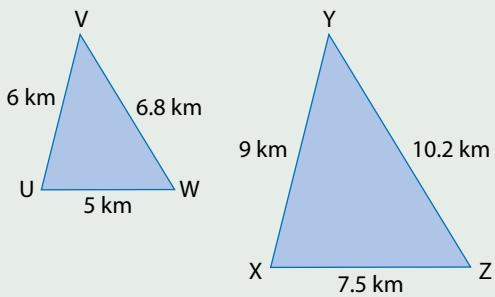
$\frac{h}{m} = \frac{17.5}{7}$ $\quad \frac{k}{p} = \frac{10}{4}$ $\quad \frac{j}{n} = \frac{15}{6}$

$\frac{h}{m} = 2.5$ $\quad \frac{k}{p} = 2.5$ $\quad \frac{j}{n} = 2.5$

Since the ratios of corresponding sides are equal, △MNP ~ △HJK.

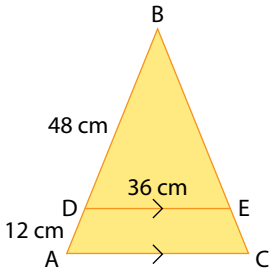
Show You Know

Show that △UVW ~ △XYZ.



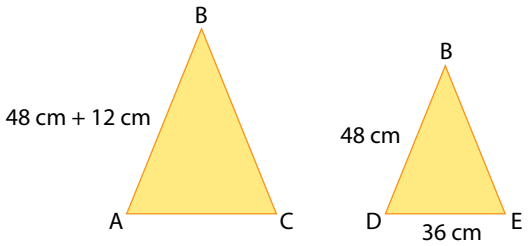
Example 3: Determine Unknown Measures in Nested Similar Triangles

In the diagram, DE is parallel to AC. Find the length of AC.



Solution

Draw the triangles separately.
AD = 12 cm and DB = 48 cm.
So, AB is the sum of the two lengths.
AB = 60 cm.



Since AC is parallel to DE, $\angle A = \angle D$ and $\angle E = \angle C$.

The angle at B is common to both triangles.

When a straight line intersects two parallel lines, corresponding angles are equal.

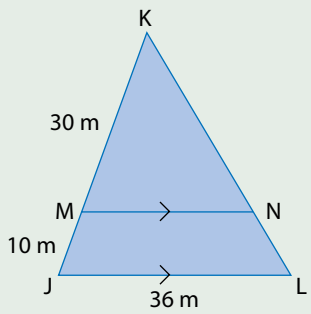
Since the two triangles have corresponding angles that are equal, $\triangle ABC \sim \triangle DBE$ and the lengths of the corresponding sides must be proportional.

$$\frac{AC}{DE} = \frac{AB}{DB}$$
$$\frac{AC}{\mathbf{36}} = \frac{\mathbf{60}}{\mathbf{48}}$$
$$AC = \frac{36 \times 60}{48}$$ Multiply both sides by 36.
$$AC = 45$$

The length of AC is 45 cm.

Show You Know

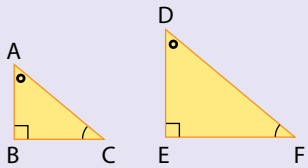
In the diagram below, MN is parallel to JL. Determine the length of MN.



Connect and Reflect

Key Ideas

- When two triangles are similar, their corresponding angles are equal and their corresponding sides are proportional.
- To prove that two triangles are similar, you need to show that either the corresponding angles are equal or that the corresponding sides are proportional



Corresponding angles are A and D, B and E, and C and F.
Corresponding sides are AB and DE, BC and EF, and AC and DF.

- To determine unknown sides in similar triangles, solve for the proportion of sides in one triangle and apply it to the second triangle, or solve for the proportion between the two triangles and apply the scale as needed.

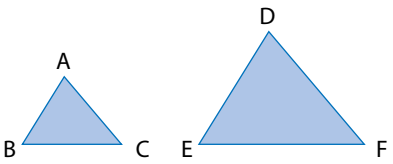
Practise

For help with #1, refer to the Explore and Analyze on page 60.

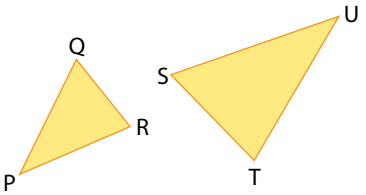
1. For each pair of similar triangles, list the corresponding angles and the corresponding sides.

How are the triangles in part a) different from those in part b)?

a)



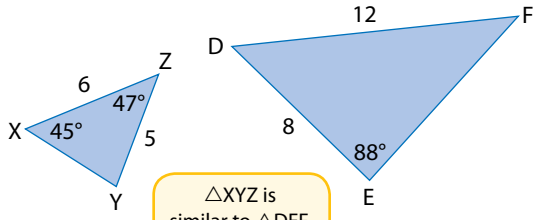
b)



For help with #2, refer to Example 1 on page 61.

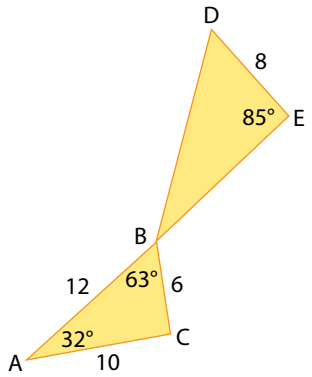
2. For each pair of similar triangles, determine all unknown angle and side length measures.

a)



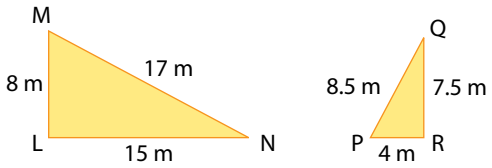
△XYZ is similar to △DEF.

b)

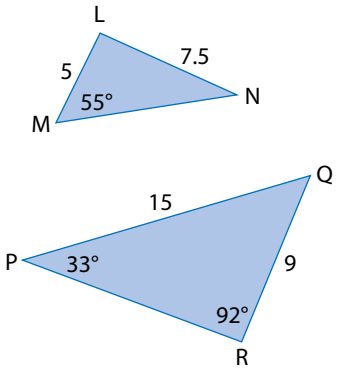


For help with #3 and #4, refer to Example 2 on page 62.

3. Show whether △LMN is similar to △PQR.

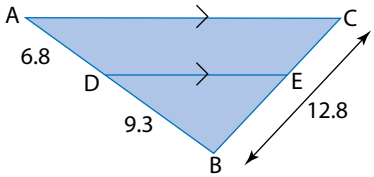


4. Show whether △LMN is similar to △PQR.

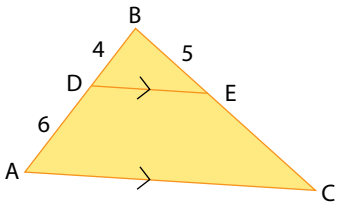


For help with #5 and #6, refer to Example 3 on page 63.

5. DE is parallel to AC, AD = 6.8, DB = 9.3, and BC = 12.8. Find the length of BE to the nearest tenth of a unit.



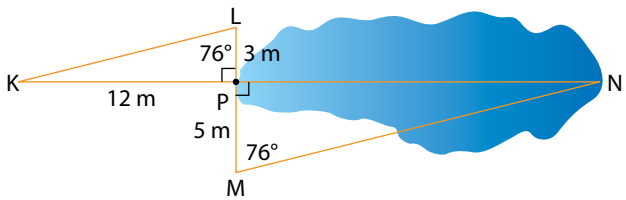
6. DE is parallel to AC, BD = 4, DA = 6, and BE = 5. Determine the length of BC.



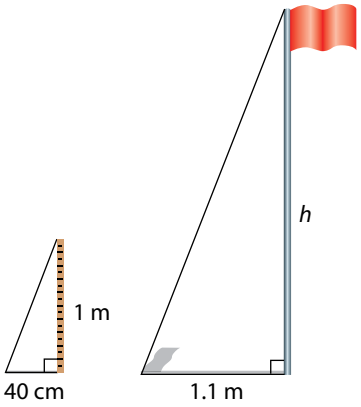
Apply

7. A pole 3 m tall casts a shadow 4 m long. At the same time, a nearby tree casts a 15-m shadow. What is the height of the tree? Discuss your reasoning with a partner.

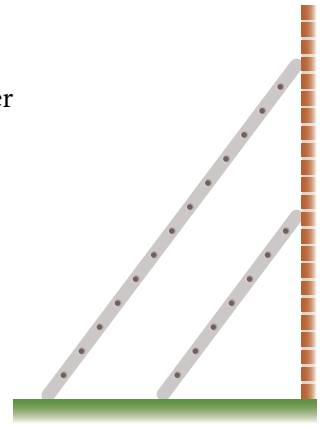
8. To find the length of a pond, a surveyor took some measurements. She recorded them on this diagram. What is the length of the pond?

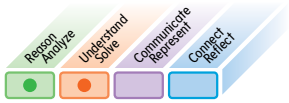


9. At noon, a metre stick casts a 40-cm shadow. At the same time, the flagpole outside the school casts a 1.1-m shadow. Apply the principles of similar triangles to find the height of the flagpole.



10. Two ladders are leaned against a wall so that they make the same angle with the ground. The 10-foot ladder reaches 8 feet up the wall. How much further up the wall does the 18-foot ladder reach?

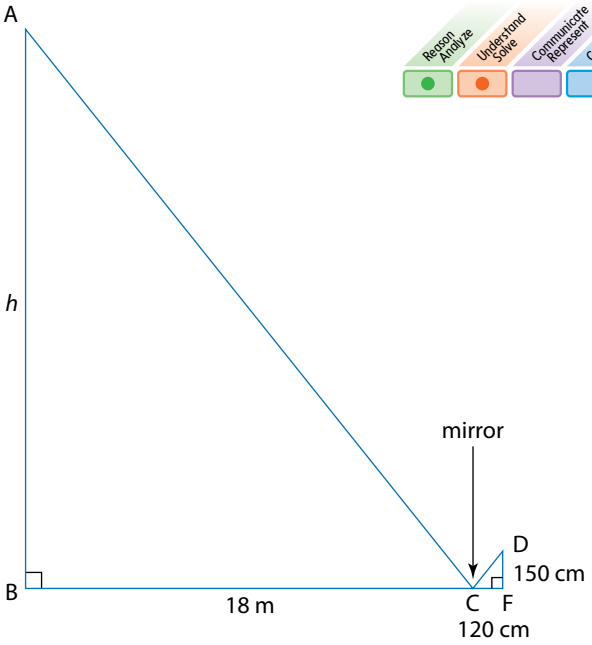




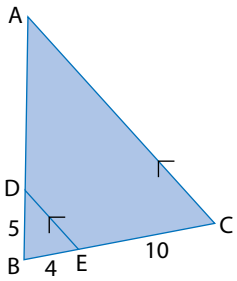
11. ✓ **Competency Check** Elizabeth's eyes are 150 cm from the floor. She places a mirror on the floor 18 m from the base of a climbing wall. She walks backward 120 cm, until she sees the top of the wall in the mirror.

a) How do you know that the two triangles are similar?

b) Determine the height of the climbing wall.



12. In the diagram, DE is parallel to AC, BD = 5, BE = 4, and EC = 10. Name the similar triangles. Determine the length of BA.

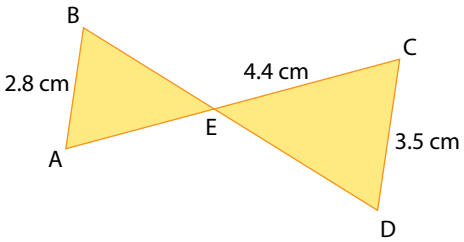


13. In the diagram, AB = 2.8 cm, CD = 3.5 cm, CE = 4.4 cm, and ∠A = ∠C.

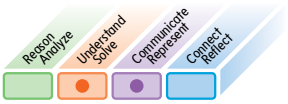
a) Name the similar triangles.

b) Explain how you know the triangles are similar.

c) Find the length of AE.



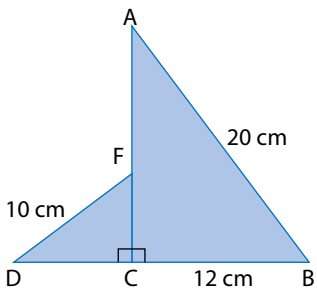
14. Design a problem that uses similar triangles that would allow you to determine the height of something that you cannot reach or measure.



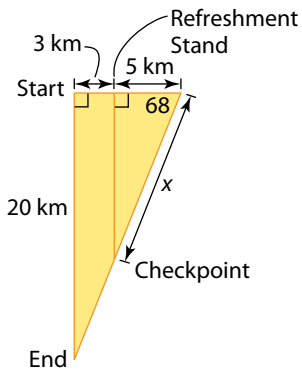
Extend

15. A ski tow rises 39.5 m over a horizontal distance of 118.8 m. Assuming this relationship stays constant, what vertical distance do you rise if you travel 750.2 m horizontally? Justify your reasoning.

16. In the diagram, ∠D = ∠A, AB = 20 cm, CB = 12 cm, and DF = 10 cm. What is the measure of AF?



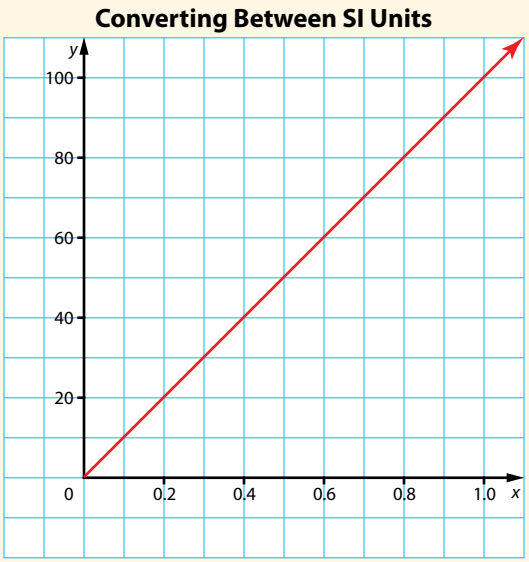
17. Orienteering is an important skill to learn in wilderness training. In one exercise, students walk due east of the starting point. There is a refreshment stand at the 3-km checkpoint. The next checkpoint is 5 km due east of the first. From there, students must walk S 22° W to the third checkpoint. In total, how far must the students walk to reach the third checkpoint?



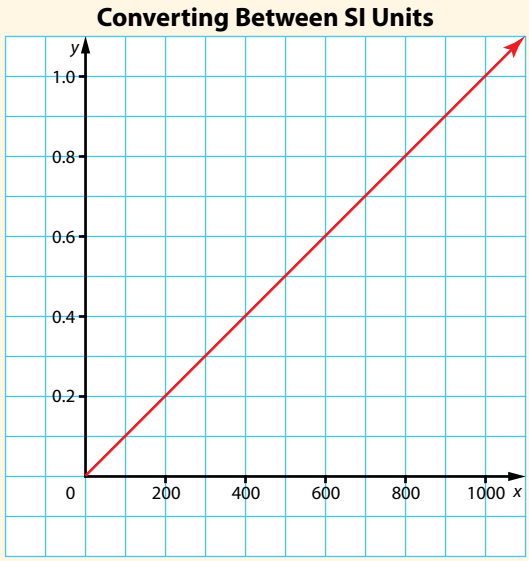
Rich Problems

1. For the graphs below, state two possible SI units that could be labelled on each axis.

a)



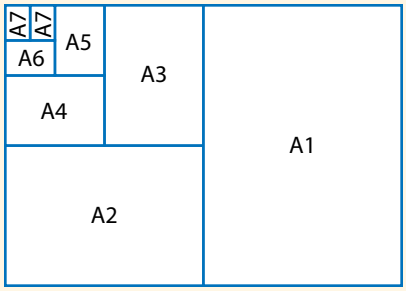
b)



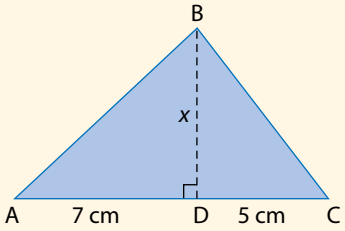
2. The international paper size standard is defined as follows:

- The A0 or starting size has an area of 1 m^2.
- The length to width ratio is $\sqrt{2}$.
- Each subsequent size A(n) is defined as A($n - 1$) cut in half parallel to its shorter side.
- The standard length and width of each size is rounded to the nearest millimeter.

If A0 has dimensions 1189 mm by 841 mm, what are the dimensions for A1, A4, and A6 sheets of paper?



3. The geometric mean between two numbers, a and b, is the value x such that $\frac{a}{x} = \frac{x}{b}$.
You are given $\triangle ABC$ such that $\triangle ADB \sim \triangle BDC$ and $\angle ADB = 90°$.



a) Why can BD be defined as the geometric mean of sides AD and DC?

b) Write a proportion and solve for x.

Chapter 2 Review

Learning Goals

Inquire and Explore: How can you describe, measure, and compare spatial relationships?
How are similar shapes related?
What characteristics make shapes similar?

After this section, I can	
2.1	■ solve problems using a personal referent to estimate measurements ■ convert between SI measurements
2.2	■ analyze to determine the scale factor by which an item was reduced or enlarged ■ solve for actual measurements using a given scale
2.3	■ represent a place or object by creating a scale diagram ■ use scale diagrams to determine unknown measurements
2.4	■ analyze to determine whether two triangles are similar ■ use similarity between triangles to solve problems

2.1 SI Measurement, pages 40–45

1. Use your collection of SI measurement referents to estimate each measure in your classroom. Justify your choice of unit.

a) the width of the door

b) the thickness of the door

c) the height of a desk or table

d) the width of the chalkboard

2. Measure each distance in #1 and compare the measurement to your estimate.

3. A ramp is 9.4 m long. How many classroom door heights would it take to exceed the length of the ramp?

4. Convert each measurement to the unit specified.

a) 17 cm = ■ mm

b) 5.5 m = ■ cm

c) 10 km = ■ m

d) 25 mm = ■ cm

5. a) Estimate the perimeter of the figure below. Use an appropriate SI unit.

b) Determine the perimeter. Compare the measurement to your estimate. How close was your estimate?

2.2 Enlargements and Reductions, pages 46–51

6. State whether each scale is an example of a scale reduction or an enlargement. Explain how you know.

a) 50 : 1

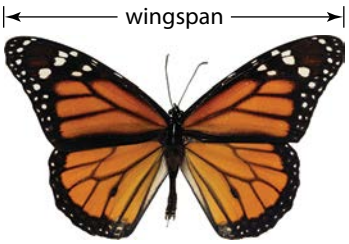
b) 1 : 50

7. Calculate the actual length given the scale and the length on a scale model.

a) 2.5 cm on a 1 : 48 model

b) 48 mm on an 8 : 1 photograph

8. Determine the scale of this photo of a monarch butterfly. The average wingspan of a monarch butterfly is about 9 cm. Express the scale as a ratio and as a fraction.

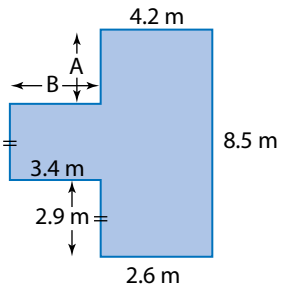


9. The most famous aircraft designed and built in Canada was the Avro Arrow. A number of $\frac{1}{8}$ scale models were made and tested in a wind tunnel. The length of each model was 10′8″. What was the length of the full-size Arrow?

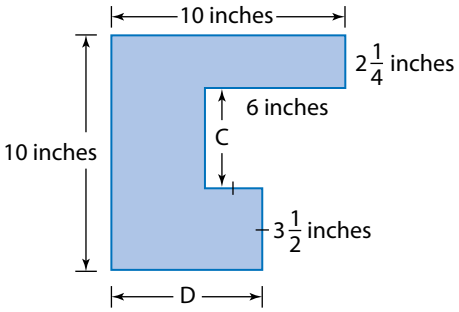
2.3 Scale Diagrams, pages 52–59

10. Determine the missing dimensions.

a)



b)



11. Calculate the perimeters of the figures in #10.

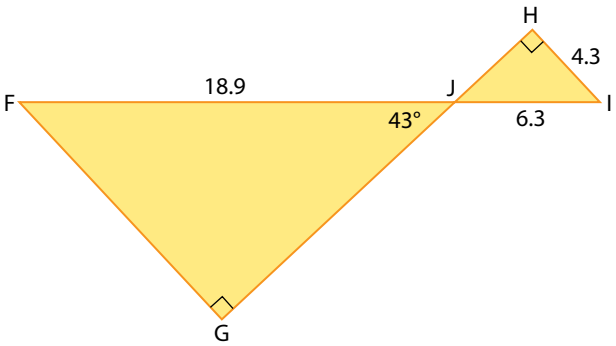
12. a) Use a scale of 1 square edge to 2 cm to create a scale drawing of the front of the cereal box shown. The actual dimensions of the box are 28 cm by 20 cm.

b) Use the same scale to draw the side of the box. The width of the actual box is 7 cm.

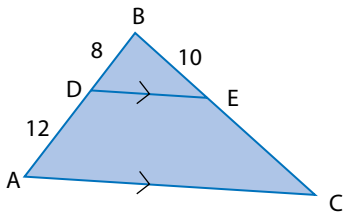
2.4 Similar Triangles, pages 60–67

13. a) For the pair of similar triangles below, list the corresponding angles and the corresponding sides.

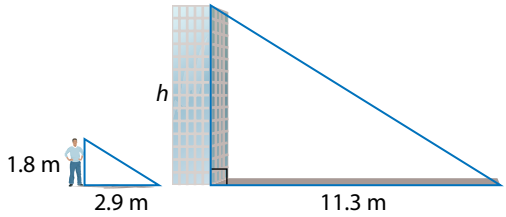
b) Determine all unknown side lengths to the nearest tenth of a unit and determine all unknown angles.



14. DE is parallel to AC, BD = 8, DA = 12, and BE = 10. Find the length of BC.

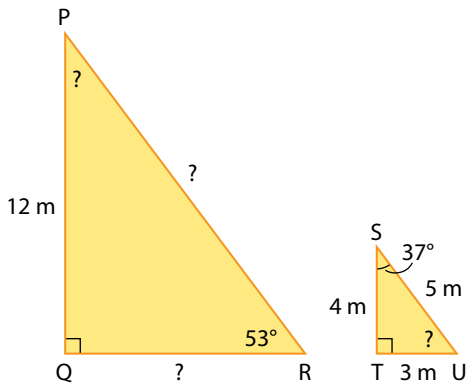


15. On a sunny day, José's shadow is 2.9 m long, while the shadow of a tower is 11.3 m long. José is 1.8 m tall. Calculate the height of the tower.



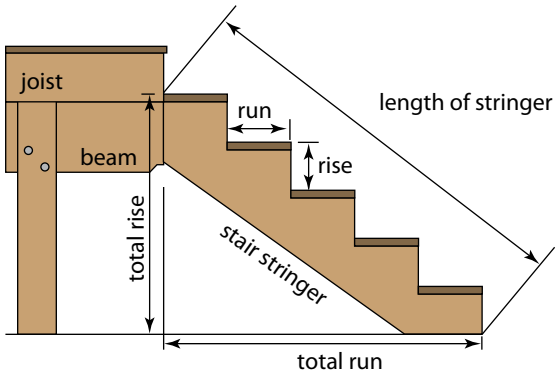
16. a) Are △PQR and △STU similar?

b) Determine the unknown side lengths and angles.



Connect the Concepts

17. A carpenter is building a set of stairs. The staircase has five steps. Each step has a rise of 18 cm and a run of 24 cm.



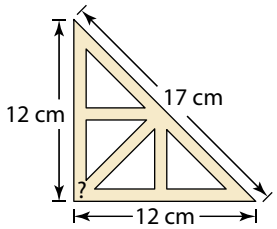
a) What is the total rise of the staircase?

b) What is the total run of the staircase?

c) Use the Pythagorean relationship to calculate the length of a piece of wood needed to make the stringer.

d) Determine the scale of the drawing.

18. A carpenter's square is a tool in the shape of a right angle. Use the drawing below to answer the questions.



a) Is the carpenter's square a right triangle? Explain your reasoning.

b) What is the scale of the drawing?

19. a) Draw a triangle on grid paper.

b) Double the dimensions, and draw a new triangle, similar to the first.

c) If you were to use a scale of 1:5, what would your triangle's new dimensions be?

d) Have a partner draw a reduction of your triangle from part a). Determine the scale factor your partner used.